Information Flow in Neural Circuits

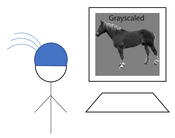
Praveen Venkatesh, Sanghamitra Dutta and Pulkit Grover

Dept. of Electrical and Computer Engineering, and the Center for the Neural Basis of Cognition, Carnegie Mellon University

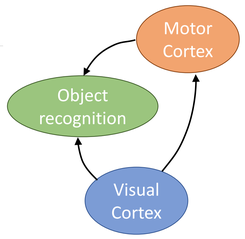
Contact: vpraveen@cmu.edu

What do we Want to Measure?

Information Paths

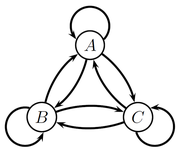


(Almeida et al., 2013)

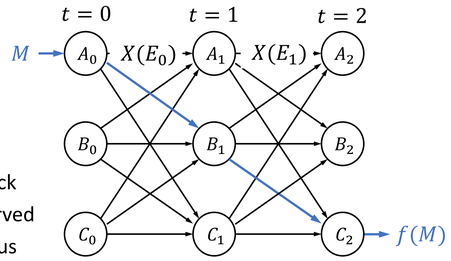


- Info flow between brain regions
- Info about a stimulus
- Feedback

A Computational Model



Brain areas, with feedback
Edges' transmissions observed
Message M is the stimulus

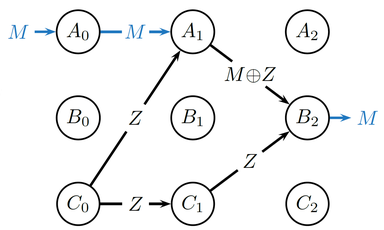


In Search of a Definition

Candidate Definition I: Mutual Information

Information flows on an edge E_t if its transmission depends on M

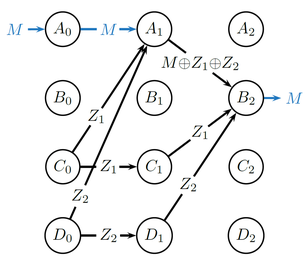
$$I\big(M; X(E_t)\big) > 0$$

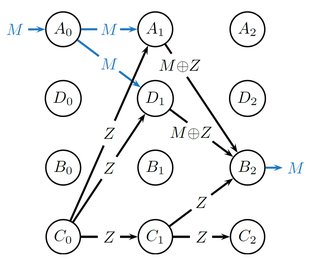


Candidate Definition II: Conditional Mutual Info

Conditioning on the other edge (Z) reveals the information flow!

$$I\big(M; X(E_t)\big) > 0 \text{ or } I(M; X(E_t) \mid X(E_t')) > 0$$





Final Definition

Condition on a *subset* of edges

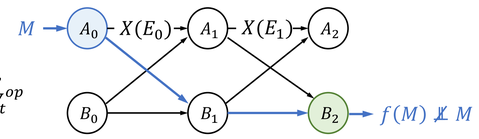
Information flows on an edge E_t if $\exists\ \mathcal{E}_t' \subseteq \mathcal{E}_t$ s.t. $I(M; X(E_t) \mid X(\mathcal{E}_t')) > 0$

Existence of Information Paths

***M*-information path:**
A path, every edge of which has M-information flow

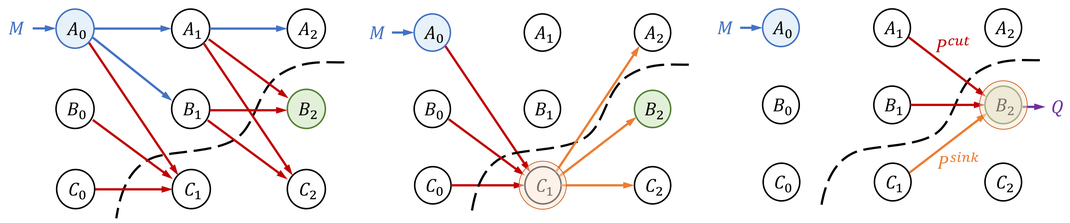
Theorem: If the transmissions of an "output" node V_t^{op} *depend* on M, then there is an M-information path leading from the input nodes to V_t^{op}

(Venkatesh et al., ISIT 2019; arXiv 2019; IEEE Trans. IT sub.)



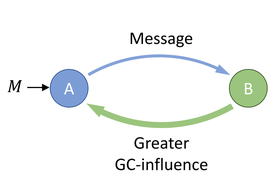
Information Paths: Proof Sketch

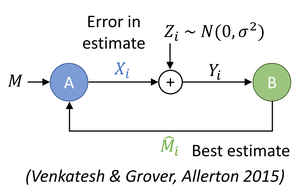
∃ *no* M-information path from V_0^{ip} to V_t^{op} ⇒ Transmissions of V_t^{op} *cannot* depend on M



$$I(M; X(Q)) \le I\left(M; X(P^{sink}), X(P^{cut})\right) = I\left(M; X(P^{sink})\right) + I\left(M; X(P^{cut}) \mid X(P^{sink})\right) = 0$$

Information Flow vs. Granger Causality





(Venkatesh & Grover, Allerton 2015)

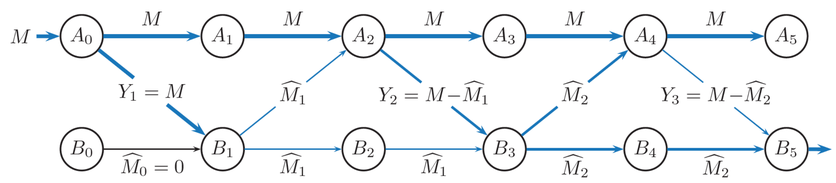
$$X_i = M - \widehat{M}_{i-1}$$

$$X_i = \sum_{j=1}^{p} \alpha_j X_{i-j} + c + \epsilon_i$$

$$X_i = \sum_{j=1}^{p} \tilde{\alpha}_j X_{i-j} + \sum_{j=1}^{q} \tilde{\beta}_j \widehat{M}_{i-j} + \tilde{c} + \tilde{\epsilon}_i$$

$$GC_{B\to A} = \frac{Var(\epsilon)}{Var(\tilde{\epsilon})} \to \infty \text{ !!}$$

Dirn. of greater GC influence can be opp. intuitive dirn. of info flow



Quantifying information flow can reveal the asymmetry between the transmitter and the receiver

(Venkatesh et al. arXiv 2019; IEEE Trans. IT sub.)

Partial Information Decomposition (PID)

$$I\big(M:(X,Y)\big) = UI(M:X \setminus Y) + UI(M:Y \setminus X) + RI(M:X;Y) + SI(M:X;Y)$$

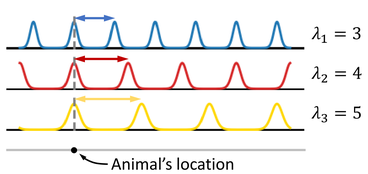
Unique to X | Unique to Y | Redundant | Synergistic

(Williams and Beer, 2010; Bertschinger et al., 2014; Schneidman et al., 2003)

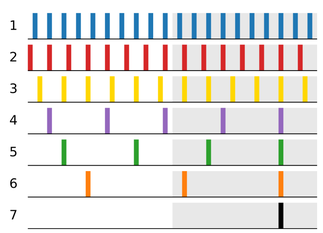
PID in *Grid Cells*

Neurons used in spatial navigation; use a robust "modulo code" to represent information about animal location

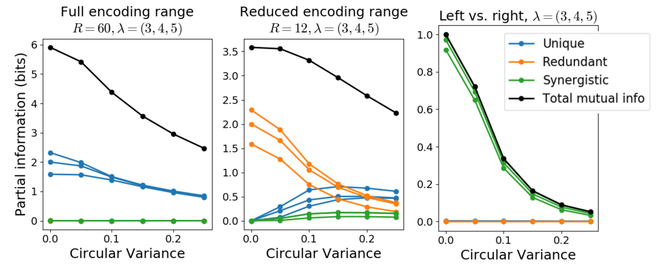
Information about animal location can be encoded uniquely, redundantly or synergistically, depending on whether or not error-correction is in effect



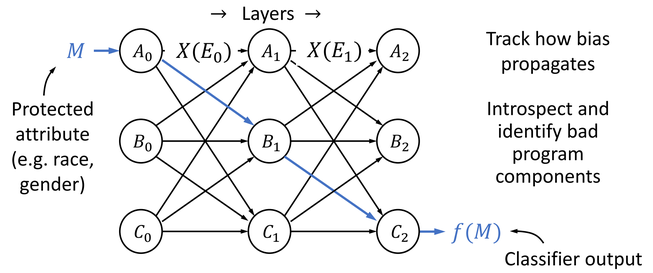
The PID provides fine-grained inferences on information flow



(Venkatesh and Grover, Cosyne 2020, accepted)



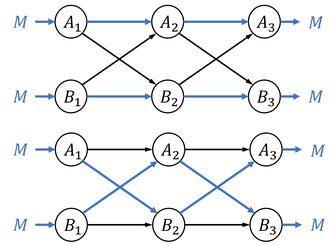
AI: Fairness and Explainability



(Novel measures of bias: Dutta, Venkatesh et al., AAAI 2020)

Implications for Neuroengineering

Can we measure only nodes? (Assume nodes multicast)



Can't differentiate these two cases

Interventions will produce different effects!

⇒ We *need* to measure edges!

Theory can give rise to new insights and problems in neuroengineering

(Patolsky et al., Science 2006)

Acknowledgments

Praveen Venkatesh was supported in part by a CIT Dean's Tuition Fellowship, a Henry L. Hillman Presidential Fellowship, the Dowd Fellowship from the College of Engineering and a Fellowship in Digital Health from the Center for Machine Learning and Health at Carnegie Mellon University.

References

1. P. Venkatesh, S. Dutta and P. Grover, "Information Flow in Computational Systems", *arXiv:1902.02292 [cs.IT]*, February 2019; *IEEE Trans. IT (submitted)*.
2. P. Venkatesh, S. Dutta and P. Grover, "How should we define Information Flow in Neural Circuits?", *ISIT*, July 2019.
3. P. Venkatesh and P. Grover, "Is the direction of greater Granger causal influence the same as the direction of information flow?", *Allerton*, September 2015.
4. P. Venkatesh and P. Grover, "Understanding Encoding and Redundancy in Grid Cells using Partial Information Decomposition", *Cosyne*, March 2020 *(accepted)*.
5. S. Dutta, P. Venkatesh, P. Mardziel, A. Datta and P. Grover, "An Information-theoretic Quantification of Discrimination with Exempt Features", *AAAI*, February 2020.
6. S. Dutta, P. Venkatesh, P. Mardziel, A. Datta and P. Grover, "Fairness under Feature Exemptions", *ISIT*, 2020 *(submitted)*.
7. N. Bertschinger, J. Rauh, E. Olbrich, J. Jost, and N. Ay, "Quantifying Unique Information", *Entropy*, 16 (4), 2161–2183, 2014.
8. E. Schneidman, W. Bialek, and M. Berry, "Synergy, Redundancy and Independence in Population Codes", *J. Neurosci.*, 27 (37), 11539–11553, 2003.
9. J. Almeida et al., "Tool manipulation knowledge is retrieved by way of the ventral visual object processing pathway", *Cortex*, 49 (9), 2334–2344, 2013.
10. F. Patolsky et al., "Detection, Stimulation and Inhibition of Neuronal Signals with High-density Nanowire Transistor Arrays", *Science*, Vol. 313, 1100–1104, 2006.