Understanding Encoding and Redundancy in Grid Cells using the Partial Information Decomposition

Praveen Venkatesh and Pulkit Grover

Dept. of Electrical and Computer Engineering, and the Center for the Neural Basis of Cognition, Carnegie Mellon University

Carnegie Mellon University

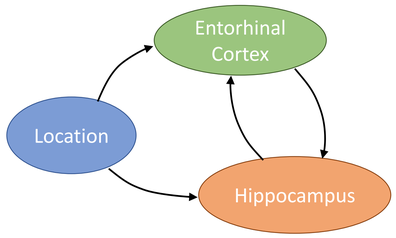
Contact: vpraveen@cmu.edu

Electrical & Computer ENGINEERING

CNBC CENTER FOR THE NEURAL BASIS OF COGNITION

Information Flow and Finer Information Structure

Existing tools for information flow, such as Granger Causality and Transfer Entropy don't explain what the information is *about*, and don't describe unique and redundant components

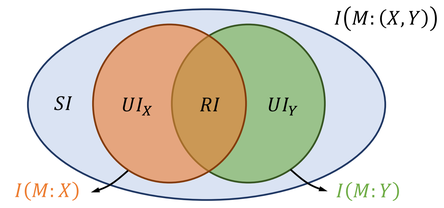


What is the Partial Information Decomposition (PID)?

An extension of mutual information to three variables: decomposing the information that X and Y carry about a message M into information that is uniquely present in each, redundantly present in both, and synergistic

$$I(M:(X,Y)) = UI(M:X \setminus Y) + UI(M:Y \setminus X) + RI(M:X;Y) + SI(M:X;Y)$$

Unique to X · Unique to Y · Redundant · Synergistic



(Williams and Beer, 2010; Bertschinger et al., 2014; Schneidman et al., 2003)

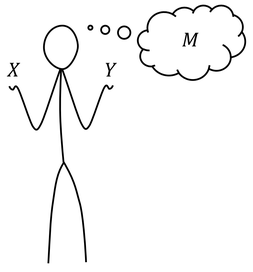
Formal Definition

There are many definitions for these quantities: we use the one of *Bertschinger et al. (2014)*:

$$UI(M:X \setminus Y) = \min_{Q \in \Delta_P} I_Q(M:X \mid Y)$$

$$\Delta_P = \{Q: Q(m,x) = P(m,x), Q(m,y) = P(m,y)\}$$

Operational meaning



Y has *no* unique information about M with respect to X if and only if you would *always* prefer to have X rather than Y to make inferences about M

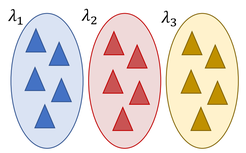
This poster uses Grid Cells as an example to motivate how the Partial Information Decomposition can provide fine-grained inferences about information flows

Also: I'm looking for a postdoc position!

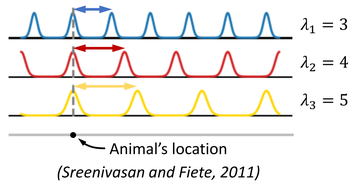
Case study: Grid cells

Neurons used in spatial navigation; use a robust "modulo code" to represent information about animal location

Information about animal location can be encoded uniquely, redundantly or synergistically, depending on whether or not error-correction is in effect

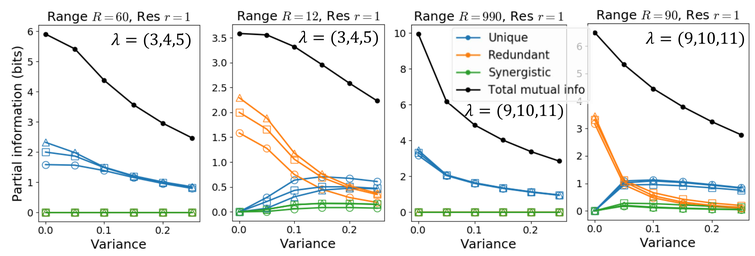


Networks with different wavelengths, each network uses a population code to encode the "phase" of the animal's location, to within this wavelength



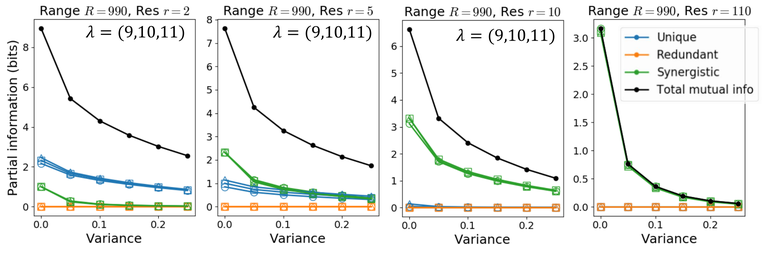
(Sreenivasan and Fiete, 2011)

Uniqueness and Redundancy

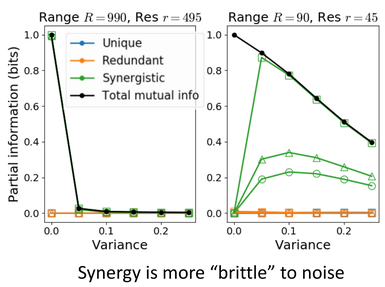


Only unique information when $R = \Pi\, \lambda_i$. Redundancy appears for a reduced encoding range

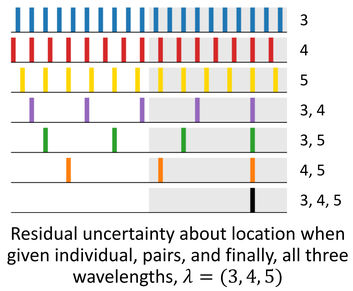
Synergy



Uniqueness decreases and synergy increases for information at coarser spatial "resolutions", r



Synergy is more "brittle" to noise



Residual uncertainty about location when given individual, pairs, and finally, all three wavelengths, $\lambda = (3,4,5)$

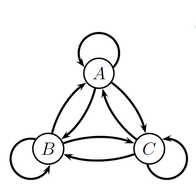
Synergy and Information Flow

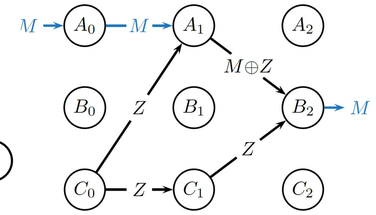
Brain areas, with feedback

Edges' transmissions observed

Message M is the stimulus

Is mutual information sufficient to capture information flow along individual edges of a computational circuit?





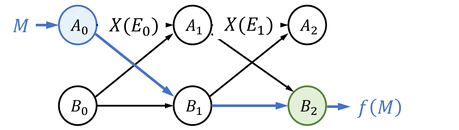
Final Definition

Condition on a *subset* of edges

Theorem

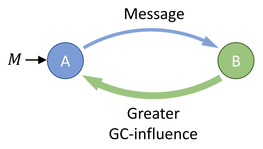
If the transmissions of an "output" node V_t^{op} *depend* on M, then there is an M-information path leading from the input nodes to V_t^{op}

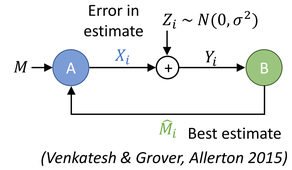
Information flows on an edge E_t if $\exists\ \mathcal{E}'_t \subseteq \mathcal{E}_t$ s.t. $I(M; X(E_t) \mid X(\mathcal{E}'_t)) > 0$



(Venkatesh et al., ISIT 2019; arXiv 2019; IEEE Trans. IT sub.)

Information Flow vs. Granger Causality





(Venkatesh & Grover, Allerton 2015)

$$X_i = M - \widehat{M}_{i-1}$$

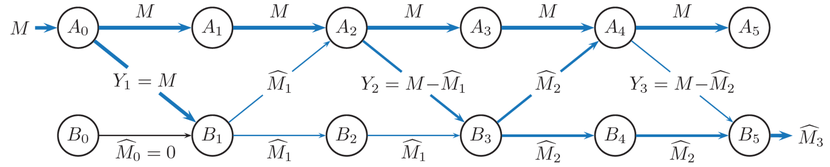
$$X_i = \sum_{j=1}^{p} \alpha_j X_{i-j} + c + \epsilon_i$$

$$X_i = \sum_{j=1}^{p} \tilde{\alpha}_j X_{i-j} + \sum_{j=1}^{q} \tilde{\beta}_j \widehat{M}_{i-j} + \tilde{\epsilon}_i$$

Dirn. of greater GC influence can be opp. intuitive dirn. of info flow

Quantifying information flow can reveal the asymmetry between the transmitter and the receiver

$$GC_{B \to A} = \frac{Var(\epsilon)}{Var(\tilde{\epsilon})} \to \infty \text{ !!}$$



(Venkatesh et al. arXiv 2019; IEEE Trans. IT sub.)

Acknowledgments

Praveen Venkatesh was supported in part by a CIT Dean's Tuition Fellowship, a Henry L. Hillman Presidential Fellowship, the Dowd Fellowship from the College of Engineering and a Fellowship in Digital Health from the Center for Machine Learning and Health at Carnegie Mellon University.

References

1. P. Venkatesh, S. Dutta and P. Grover, "Information Flow in Computational Systems", *arXiv:1902.02292 [cs.IT]*, February 2019; *IEEE Trans. IT (submitted)*.
2. P. Venkatesh, S. Dutta and P. Grover, "How should we define Information Flow in Neural Circuits?", *ISIT*, July 2019.
3. P. Venkatesh and P. Grover, "Is the direction of greater Granger causal influence the same as the direction of information flow?", *Allerton*, September 2015.
4. N. Bertschinger, J. Rauh, E. Olbrich, J. Jost, and N. Ay, "Quantifying Unique Information", *Entropy*, 16 (4), 2161–2183, 2014.
5. E. Schneidman, W. Bialek, and M. Berry, "Synergy, Redundancy and Independence in Population Codes", *J. Neurosci.*, 27 (37), 11539–11553, 2003.
6. S. Sreenivasan and I. Fiete, "Grid cells generate an Analog Error Correcting Code for Singularly Precise Neural Computation", *Nature Neuroscience*, 14 (10), 1330–1337, 2011.