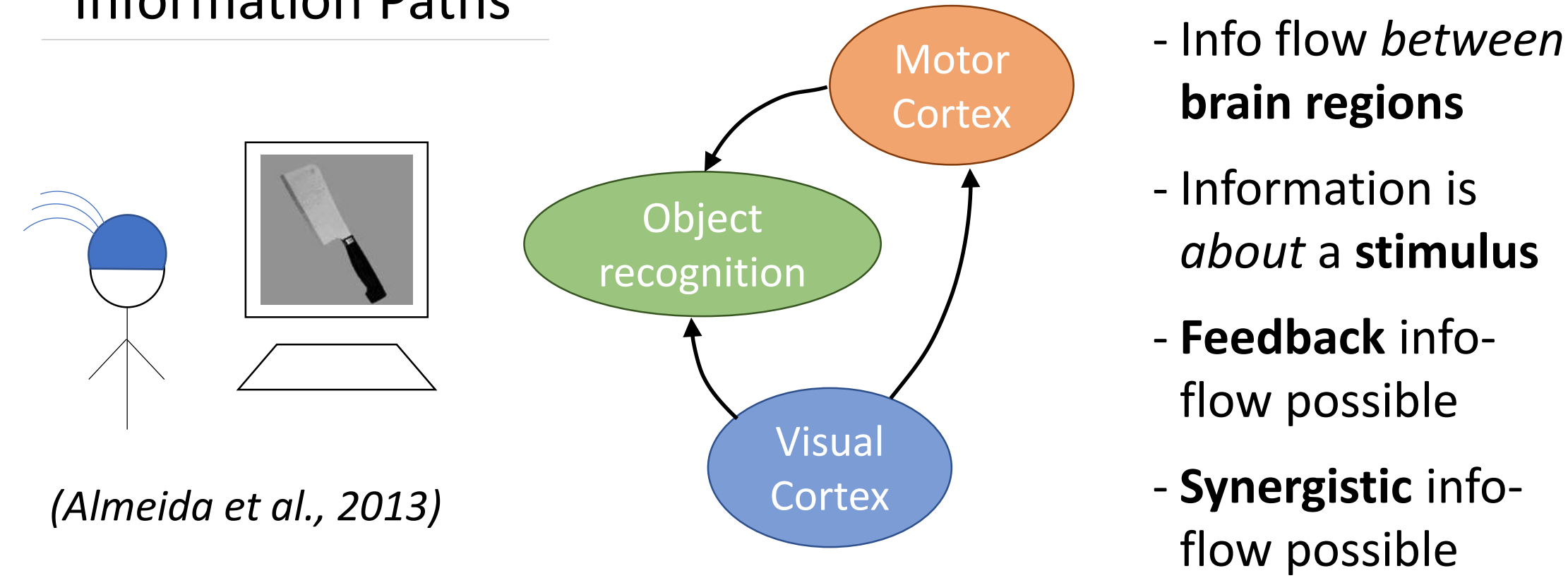


## What do we want to measure?

### Information Paths

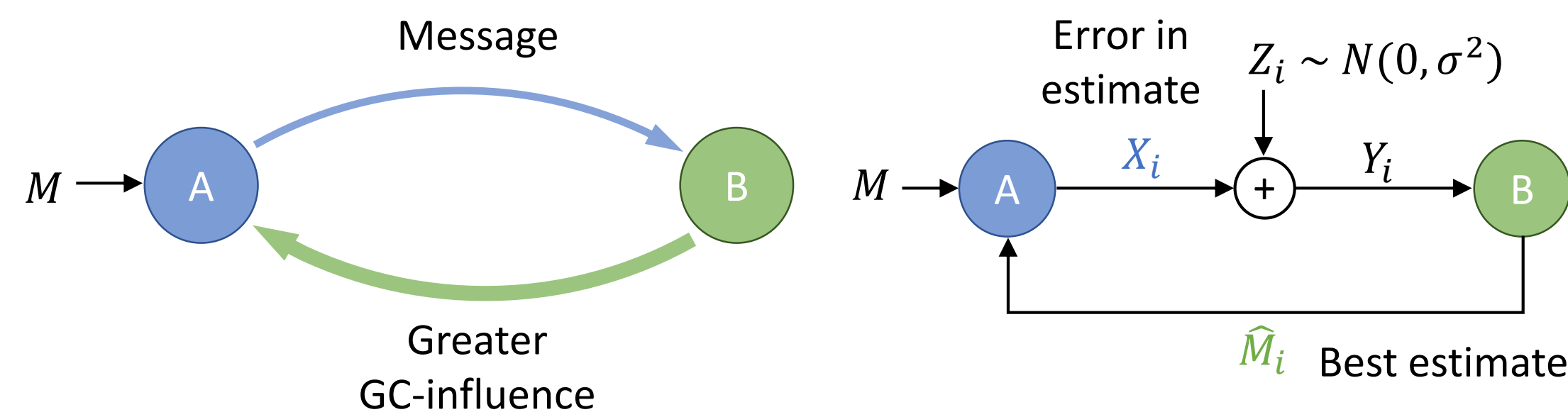


(Almeida et al., 2013)

(M.I. Posner, 1980) (Sreenivasan and Fiete, 2011) (Schneidman et al., 2003)

## Previous Approaches

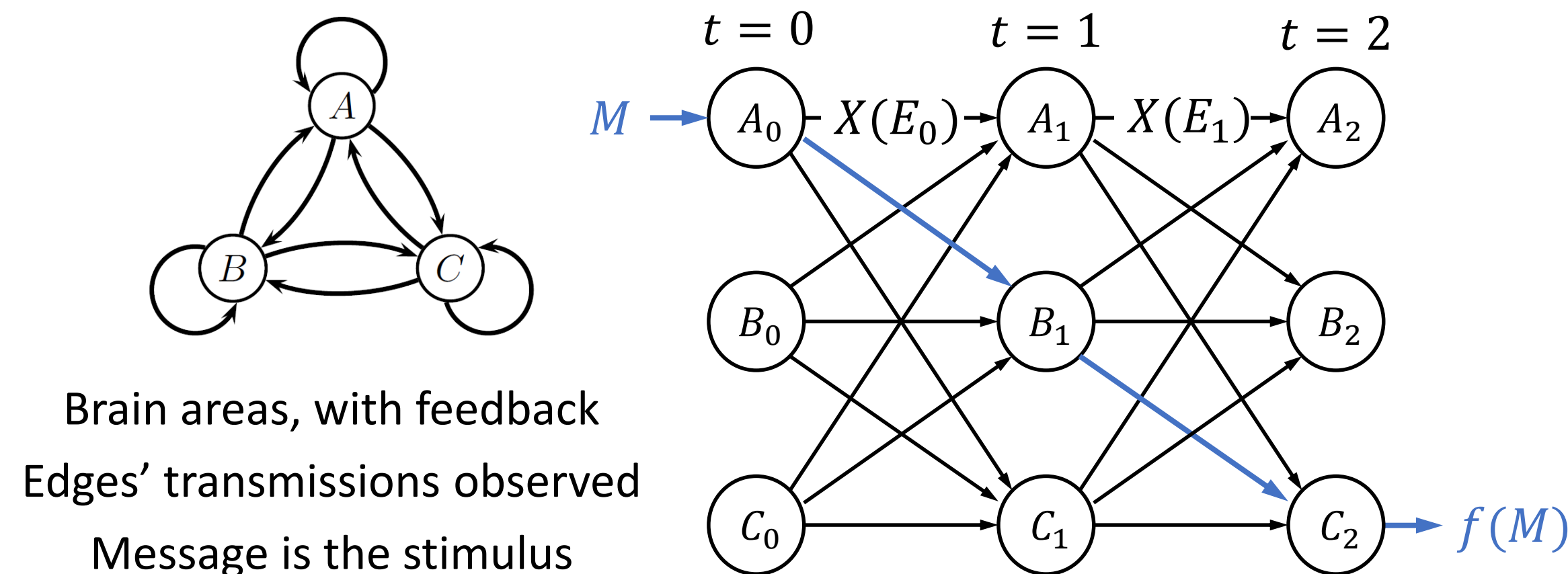
Granger Causality, Transfer Entropy, Directed Information



The direction of greater Granger causal influence can be opposite to the direction of information flow

(Venkatesh and Grover, Allerton & SfN, 2015)

## A Computational Model



(Thompson, 1980; Ahlswede et al., 2000; Peters et al., 2016)

## In Search of a Definition

**Candidate Definition I: Mutual Information**

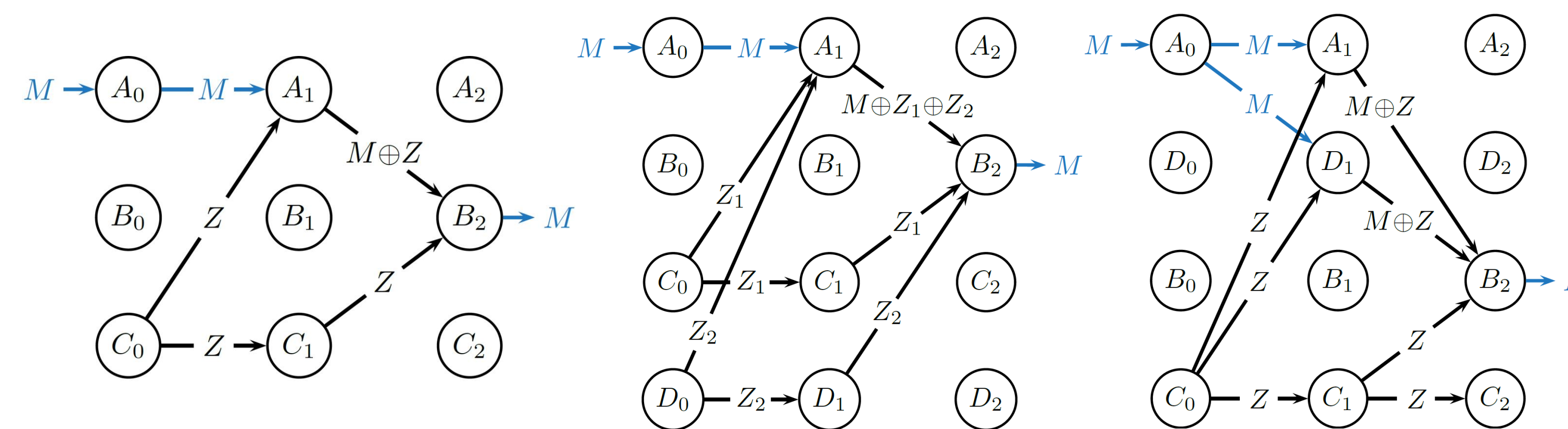
Information flows on an edge  $E_t$  if its transmission depends on  $M$

$$I(M; X(E_t)) > 0$$

**Candidate Definition II: Conditional Mutual Info**

Conditioning on the other edge ( $Z$ ) reveals the information flow!

$$I(M; X(E_t)) > 0 \text{ or } I(M; X(E_t) | X(E'_t)) > 0$$



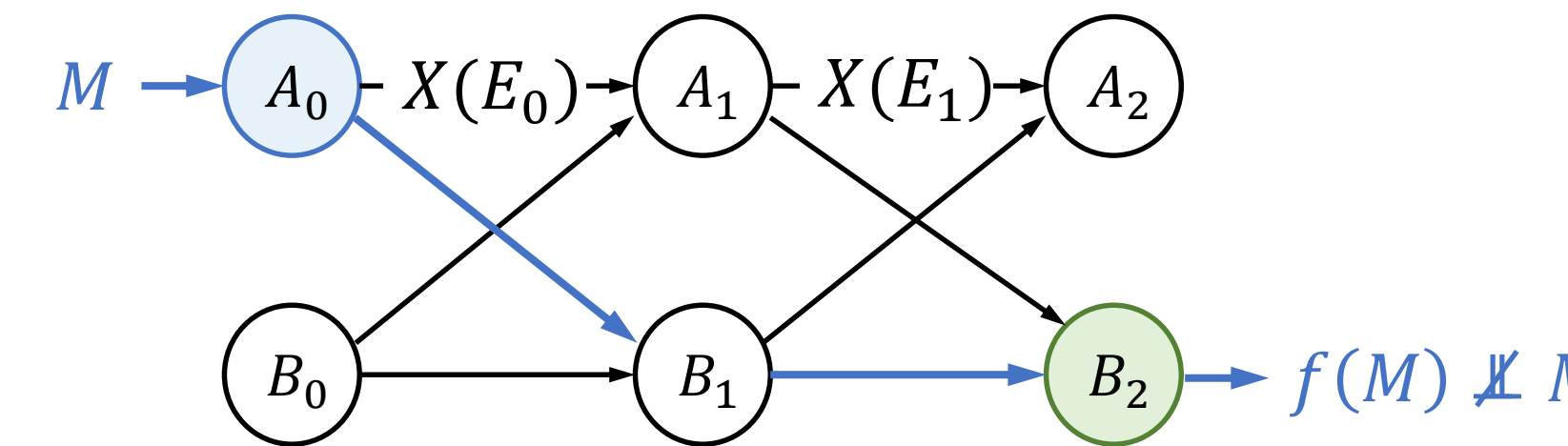
**Final Definition:** Condition on a *subset* of edges

Information flows on an edge  $E_t$  if  $\exists \mathcal{E}'_t \subseteq \mathcal{E}_t$  s.t.  $I(M; X(E_t) | X(\mathcal{E}'_t)) > 0$ .

## Information Paths

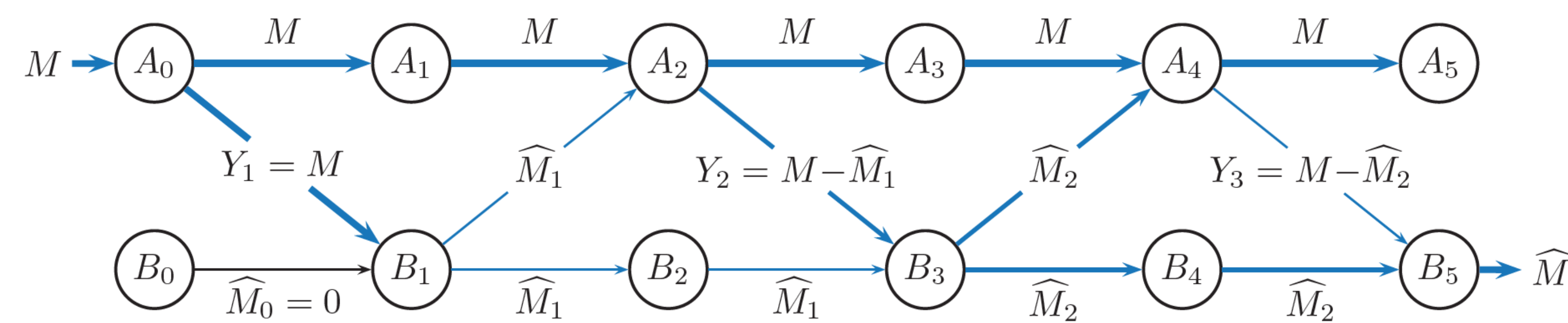
**M-information path:**  
 Every edge has  $M$ -information flow

If the transmissions of an "output" node  $V_t^{op}$  depend on  $M$ , then there is an  $M$ -information path leading from the input nodes to  $V_t^{op}$ .



## Information Flow and Feedback

Quantifying information flow can reveal the asymmetry between the transmitter and the receiver



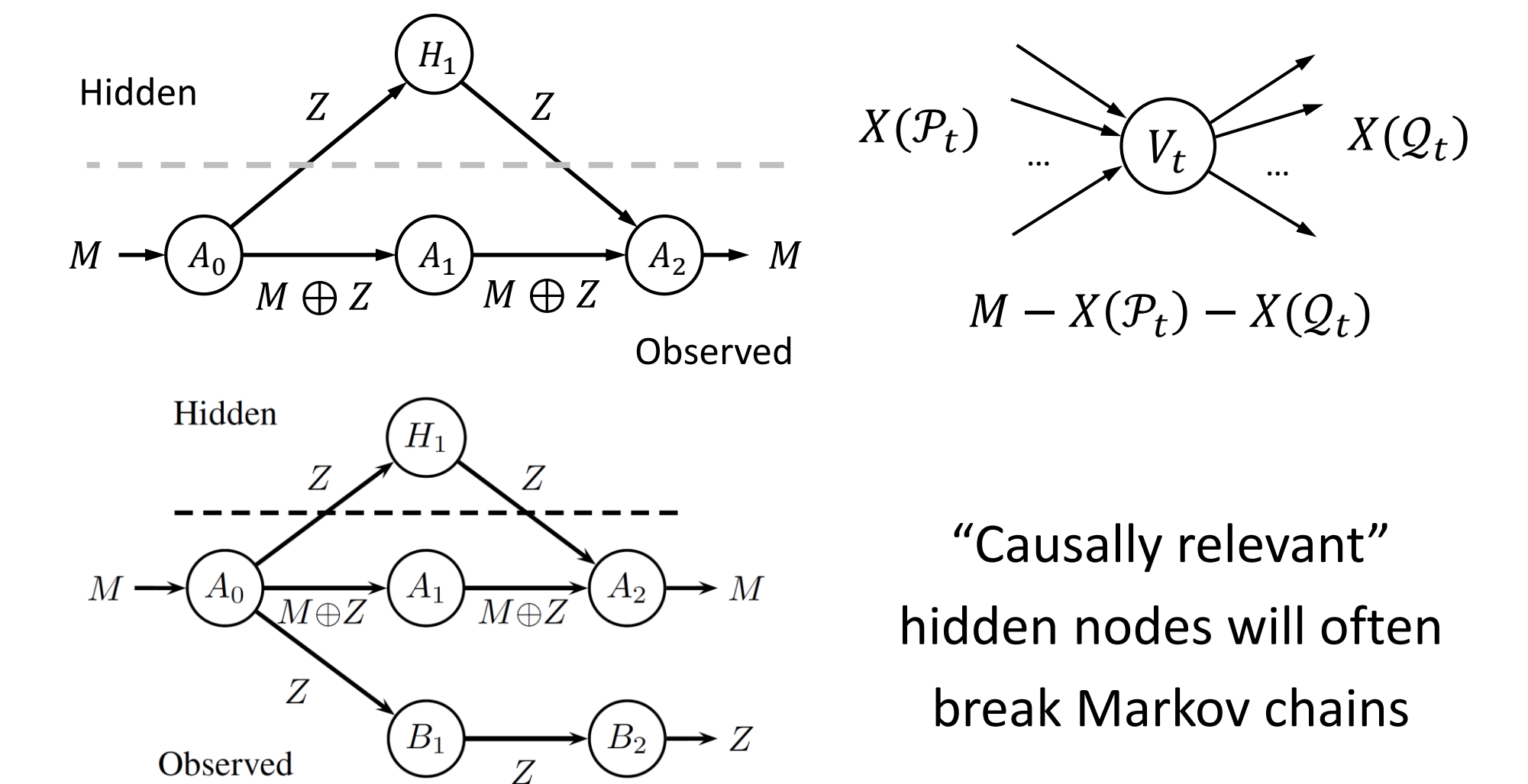
Bob's transmissions *are*  $M$ -derived from Alice's transmissions, but Alice's transmissions are *not*  $M$ -derived from Bob's transmissions

(Venkatesh et al. 2019)

✓  $M - [M, M - \hat{M}_1] - \hat{M}_2$

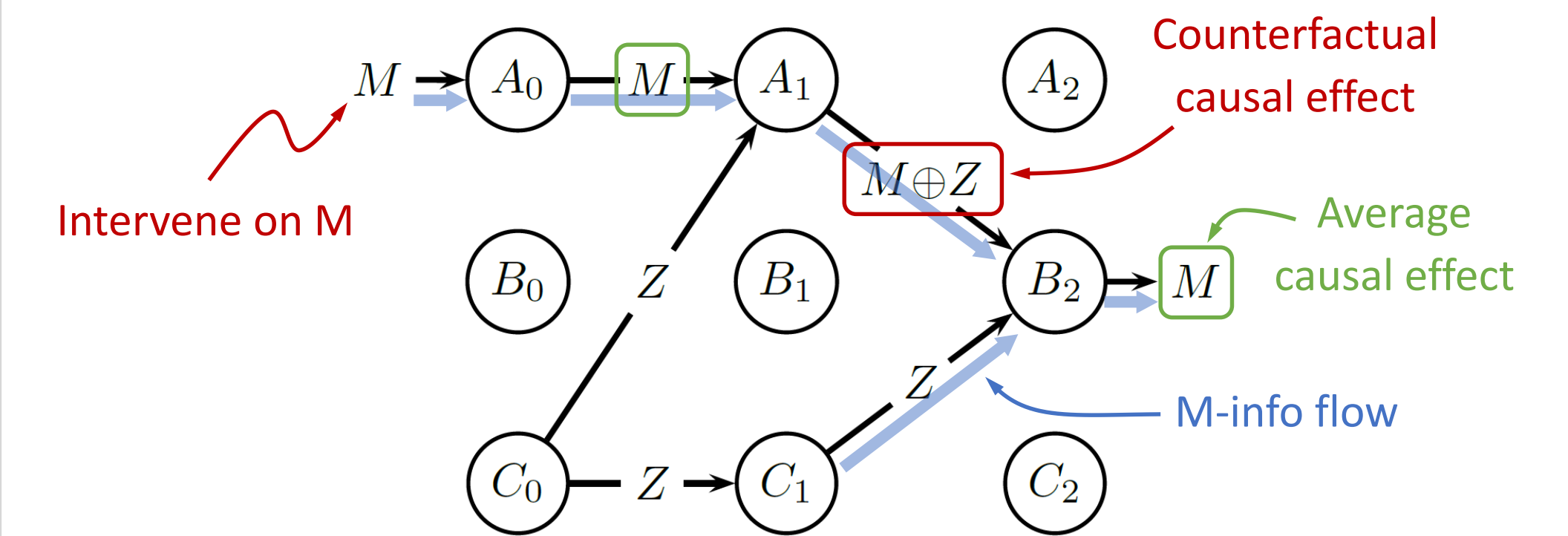
✗  $M - [\hat{M}_1, \hat{M}_2] - M - \hat{M}_2$

## Discovering Hidden Nodes



## Causal Effect and Info Flow

Average causal effect at a node's output  $\Rightarrow$  M-dependence  $\Rightarrow$  M-information flow



Some counterfactual causal effects are captured by M-information flow: even if there is *no* average causal effect

## Acknowledgments

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## References

1. P. Venkatesh, S. Dutta and P. Grover, "Information Flow in Computational Systems", *arXiv:1902.02292 [cs.IT]*, February 2019.
2. P. Venkatesh, S. Dutta and P. Grover, "How should we define Information Flow in Neural Circuits?", *ISIT*, July 2019 (accepted).
3. P. Venkatesh and P. Grover, "Is the direction of greater Granger causal influence the same as the direction of information flow?", *Allerton*, September 2015.
4. J. Almeida et al., "Tool manipulation knowledge is retrieved by way of the ventral visual object processing pathway", *Cortex*, 49 (9), 2334–2344, 2013.
5. M. I. Posner, "Orienting of Attention", *Quart. J. of Exp. Psychol.*, 32(1), 3–25, 1980.
6. S. Sreenivasan and I. Fiete, "Grid cells generate an analog error-correcting code for singularly precise neural computation", *Nature neuroscience*, 14.10: 1330, 2011.
7. E. Schneidman, W. Bialek and M. Berry, "Synergy, Redundancy and Independence in Population Codes", *J. Neurosci.*, 23(37), 11539–11553, 2003.