# A Systematic way to Define Information Flow

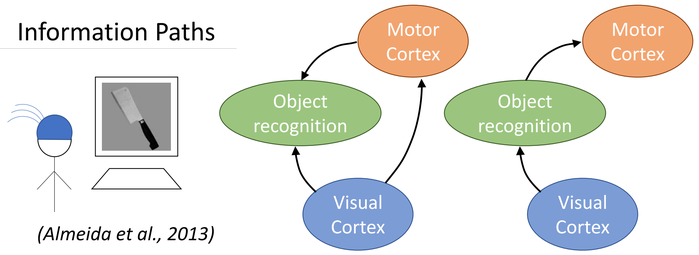
Praveen Venkatesh and Pulkit Grover

Dept. of Electrical and Computer Engineering, and the Center for the Neural Basis of Cognition, Carnegie Mellon University

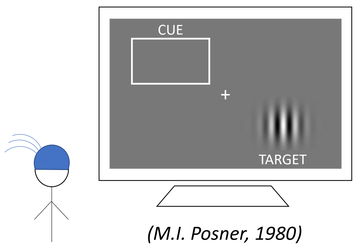
*Contact: vpraveen@cmu.edu*

## What do we want to measure?

### Information Paths

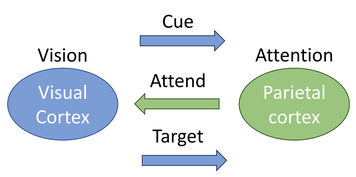



*(Almeida et al., 2013)*




*(M.I. Posner, 1980)*

### Feedback Information



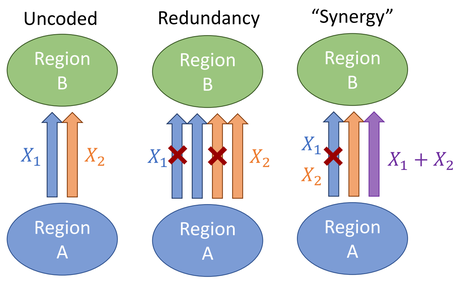

### Types of Information

Unique

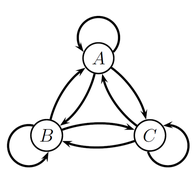
Redundant, Synergistic

Derived

*(Sreenivasan and Fiete, 2011)*



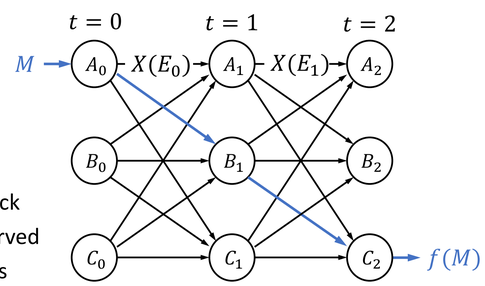

## A Computational Model




Brain areas, with feedback

Edges' transmissions observed

Message is the stimulus




*(Thompson, 1980; Ahlswede et al., 2000; Peters et al., 2016)*

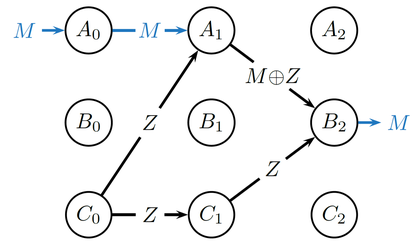
## In Search of a Definition

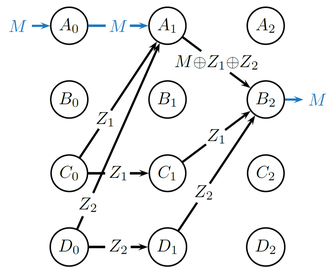
### Candidate Definition I: Mutual Information

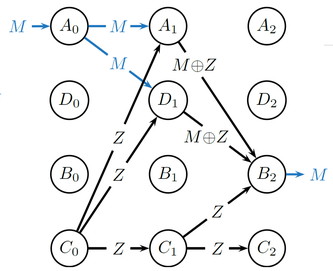
Information flows on an edge $E_t$ if its transmission depends on $M$

$$I\big(M; X(E_t)\big) > 0$$

But! In networks with synergy, mutual information fails to capture information paths




### Candidate Definition II: Conditional Mutual Info

Conditioning on the other edge (Z) reveals the information flow!

Counterexamples for conditioning on one (left), or all (right) edges

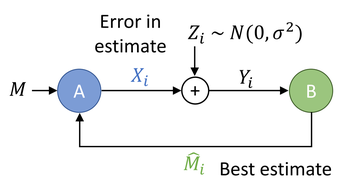
### Final Definition

Condition on a *subset* of edges

Information flows on an edge $E_t$ if, there exists some $\mathcal{E}'_t \subseteq \mathcal{E}_t$ s.t.

$I(M; X(E_t) \mid X(\mathcal{E}'_t)) > 0.$

## Granger Causality and Information Flow

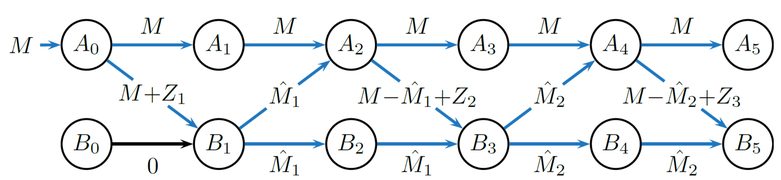



*(Venkatesh and Grover, 2015)*

The direction of greater Granger causal influence can be opposite to the direction of information flow

Derived Information can reveal the asymmetry between the transmitter and the receiver

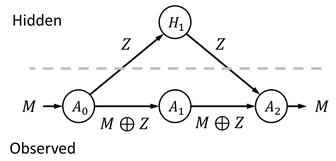
$M$—$X(P_{t'})$—$X(Q_t)$

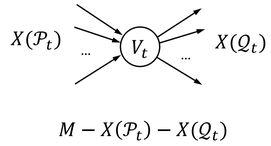



Bob's transmissions *are* M-derived from Alice's transmissions,

but Alice's transmissions are *not* M-derived from Bob's transmissions

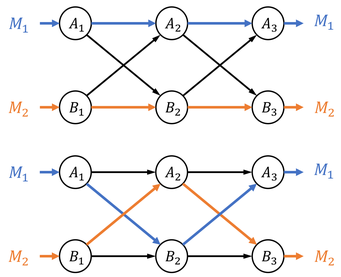
## Discovering Hidden Nodes



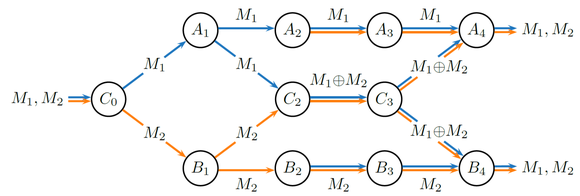

Relevant hidden nodes will often break Markov chains

## Information Flow in ANNs




We can distinguish how information flows in small toy neural networks, which have known ground truth

## Multiple Messages




*(Ahlswede et al., 2000)*

## Acknowledgments

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## References

1. P. Venkatesh, S. Dutta and P. Grover, "Information Flow in Computational Systems", *arXiv:1902.02292 [cs.IT]*, February 2019.
2. P. Venkatesh, S. Dutta and P. Grover, "How should we define Information Flow in Neural Circuits?", *ISIT*, July 2019 *(accepted)*.
3. P. Venkatesh and P. Grover, "Is the direction of greater Granger causal influence the same as the direction of information flow?", *Allerton*, September 2015.
4. J. Almeida et al., "Tool manipulation knowledge is retrieved by way of the ventral visual object processing pathway", *Cortex*, 49 (9), 2334–2344, 2013.
5. M. I. Posner, "Orienting of Attention", *Quart. J. of Exp. Psychol.*, 32(1), 3–25, 1980.
6. S. Sreenivasan and I. Fiete, "Grid cells generate an analog error-correcting code for singularly precise neural computation", *Nature neuroscience,* 14.10: 1330, 2011.