Is the direction of Granger causal influence the same as the direction of information flow?

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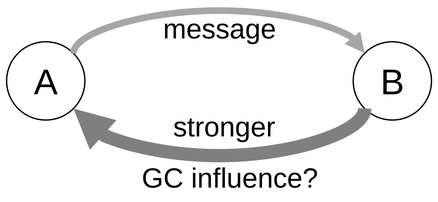
Motivation

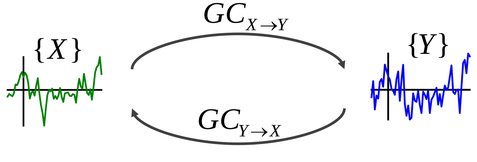
- Inferring direction of information flow is important to understand how the brain computes: the goal of the BRAIN initiative
- Granger causality (GC) is used to find direction of causal influence ("driving force") (Brovelli et. al., '04).
- **But can causal influence be interpreted as information flow?**

Can GC find the direction of information flow?

- Networks of the brain contain feedback links, with influences going both ways (Kandel, et. al., '00)
- What if part A of the brain can send a message to part B, even while B has greater Granger-causal influence on A?



Granger Causality



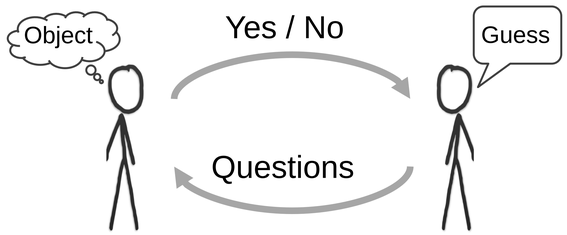
- GC can be used to interpret which process is "driving" the other, i.e., which better (causally) predicts the other (Granger '69)
- The GC-index (from X to Y) is the ratio of residual variances, $Var(\varepsilon)/Var(\tilde{\varepsilon})$

$$Y_i = \sum_{j=1}^{p} \alpha_j Y_{i-j} + \varepsilon_i$$

$$Y_i = \sum_{j=1}^{p} \alpha_j Y_{i-j} + \sum_{j=1}^{p} \beta_j X_{i-j} + \tilde{\varepsilon}_i$$

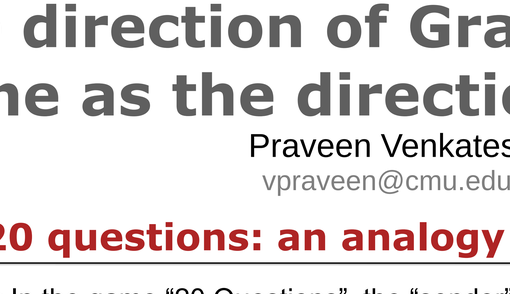
20 questions: an analogy

- In the game "20 Questions", the "sender" thinks of an object, while the "receiver" tries to guess the object by asking yes/no questions. (Wikipedia: Twenty questions)



- The "message": "Object" sender thought of.
- Who is really "driving" the other's transmission process?

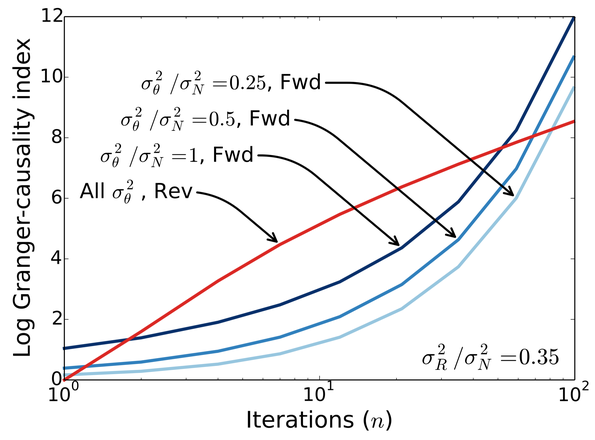
A formal counter-example



- Motivated by a capacity-achieving feedback-communication scheme (Schalkwijk et. al., '66)
- Sender wants to convey a single number "Θ" to the receiver.
- Noise in the forward link is greater than noise in the reverse link
- Communicate by feeding back the best estimate of Θ from the receiver to the sender
- Sender re-transmits error in the estimate
- Receiver updates its estimate using the new transmission, and iterates.
- Since $Var\{R\} < Var\{N\}$, the $\{\hat{\Theta}\}$ process is more predictive of the $\{X\}$ process.

Which GC-index is larger?

- Θ (the message) is going from the sender to the receiver, but the sender's transmissions are better predicted by the receiver's transmissions.
- Greater Granger-causal influence seen from the receiver to the sender.
- **The direction of Granger can be opposite to the direction of flow of the message.**

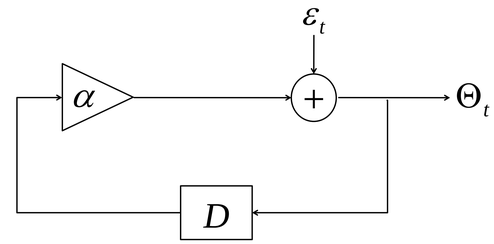


Conclusions

- Information-theoretically optimal feedback strategies exist, for which Granger-causal influence is opposite the true direction of information flow
- Previous critiques of Granger causality - the hidden node problem (Pearl, '00) and measurement noise (Friston et. al., '14) deal with incorrect estimation of the GC-index.
  - ➢ These can potentially be avoided by making better measurements.
- Our work shows: even if the GC-index is estimated perfectly, it only captures direction of causal influence, i.e., which process is driving the other.

Future work

- Evolving source model, where Θ evolves over time - all processes can be modeled as stationary AR processes.
- Corresponds better with MVAR models for Local Field Potentials.



- Finding an alternative to GC, for predicting direction of information flow. Exploit timing information - sender knows the message before the receiver.

Acknowledgements

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