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## Motivation

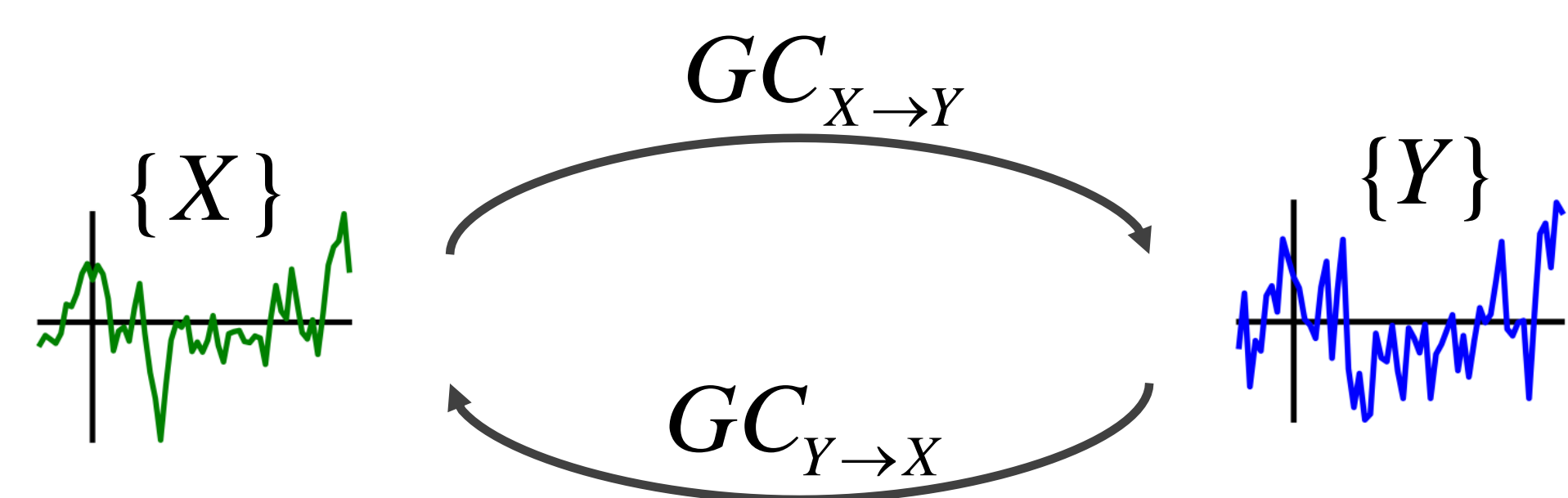
- Inferring direction of information flow is important to understand how the brain computes: the goal of the BRAIN initiative
- Granger causality (GC) is used to find direction of greater causal influence (“driving force”) (Brovelli et. al., ‘04).
- **But can greater causal influence be interpreted as information flow? (Venkatesh and Grover, ‘15)**

## Can GC comparison find the direction of information flow?

- Networks of the brain contain feedback links, with influences going both ways (Kandel et. al., ‘00)
- What if part A of the brain can send a message to part B, even while B has greater Granger-causal influence on A?
- “[Comparing GC influences] comes at the cost of a lack of sensitivity to true bi-directional interactions” (Roebroeck et. al., ‘05)



## Granger Causality



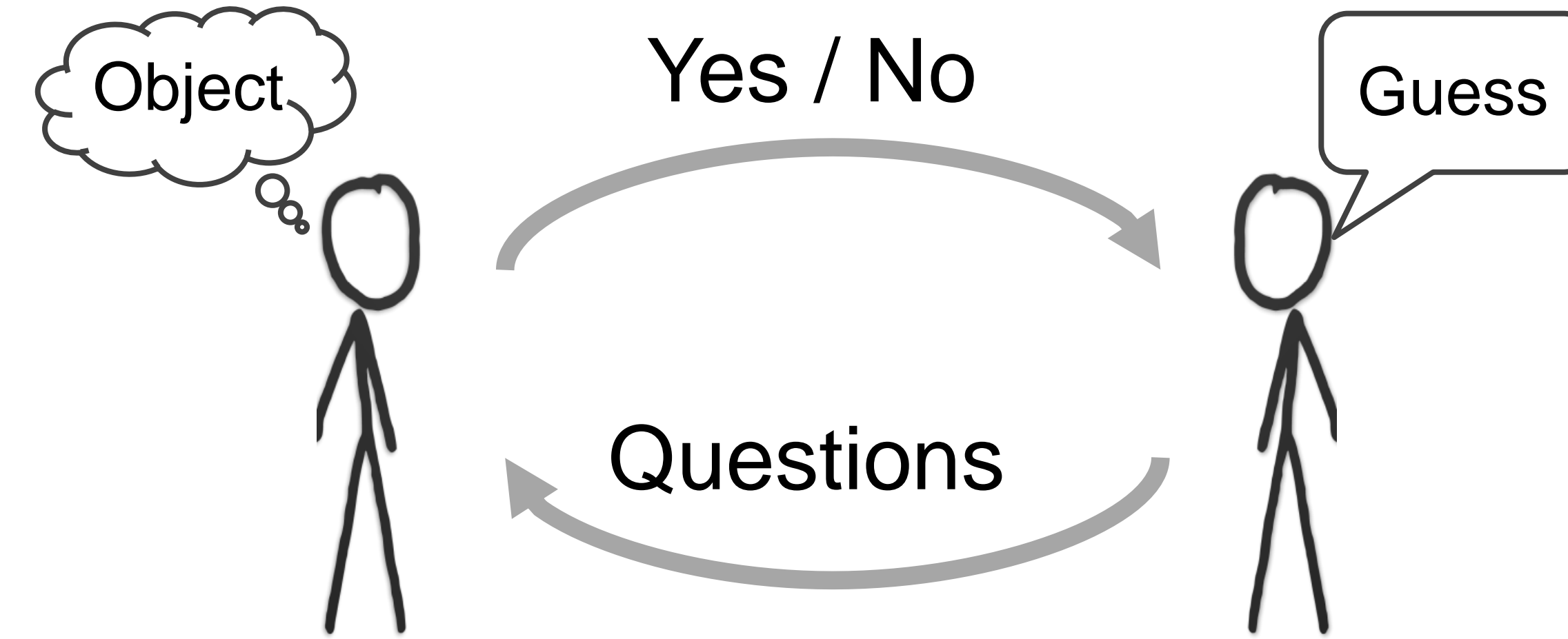
- GC can be used to interpret which process is “driving” the other, i.e., which better (causally) predicts the other (Granger ‘69)
- The GC-index (from X to Y) is the ratio of residual variances,  $Var(\varepsilon)/Var(\tilde{\varepsilon})$

$$Y_i = \sum_{j=1}^p \alpha_j Y_{i-j} + \varepsilon_i$$

$$Y_i = \sum_{j=1}^p \alpha_j Y_{i-j} + \sum_{j=1}^p \beta_j X_{i-j} + \tilde{\varepsilon}_i$$

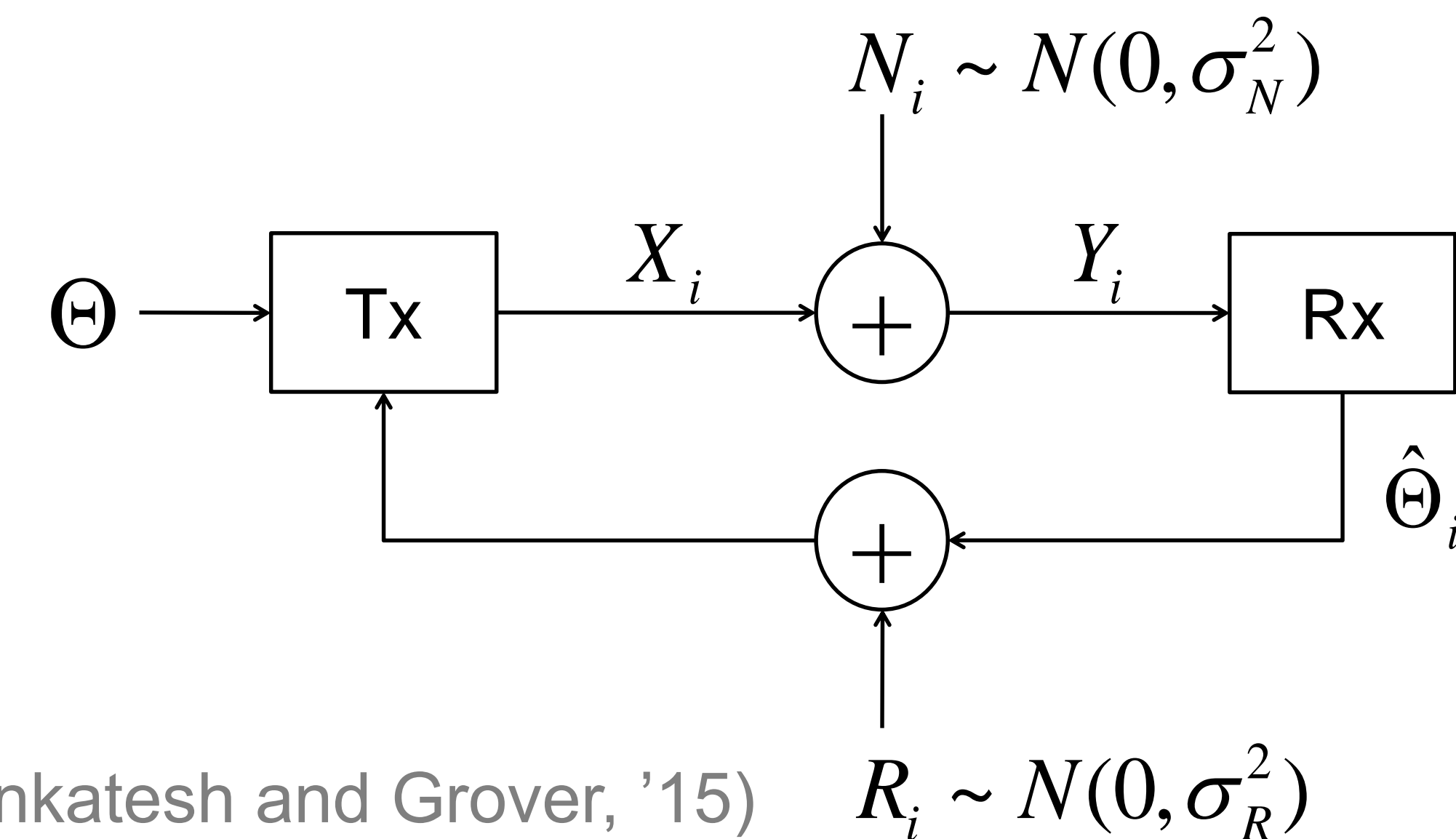
## 20 questions: an analogy

- In the game “20 Questions”, the “sender” thinks of an object, while the “receiver” tries to guess the object by asking yes/no questions. (Wikipedia: Twenty questions)



- The “message”: “Object” sender thought of.
- Who is really “driving” the other’s transmission process?

## A formal counter-example

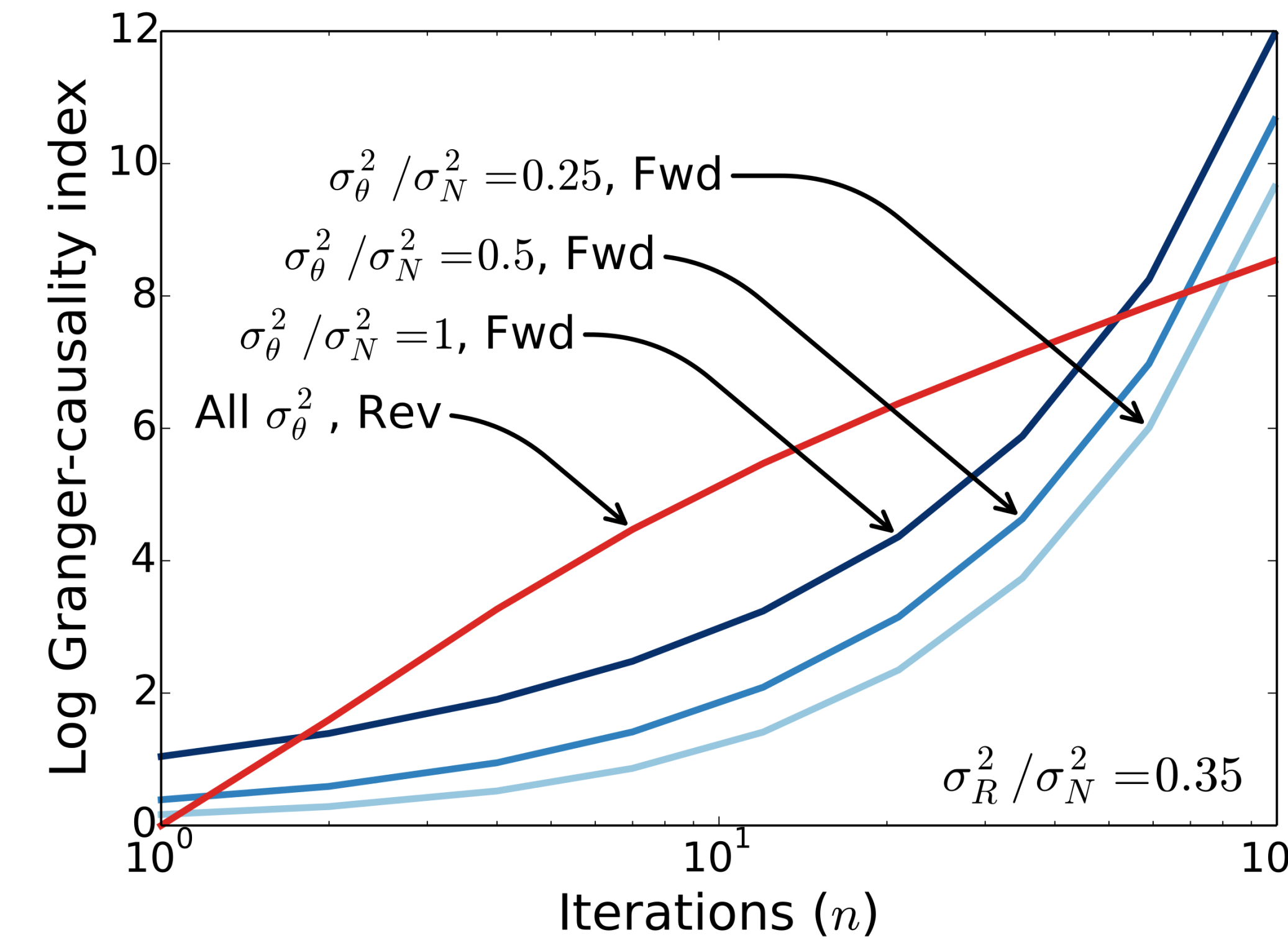


(Venkatesh and Grover, ‘15)

- Motivated by a capacity-achieving feedback-communication scheme (Schalkwijk et. al., ‘66)
- Sender wants to convey a single number “ $\Theta$ ” to the receiver.
- Noise in the forward link is greater than noise in the reverse link
- Communicate by feeding back the best estimate of  $\Theta$  from the receiver to the sender
- Sender re-transmits error in the estimate
- Receiver updates its estimate using the new transmission, and iterates.
- Since  $Var\{R\} < Var\{N\}$ , the  $\{\hat{\Theta}\}$  process is more predictive of the  $\{X\}$  process.

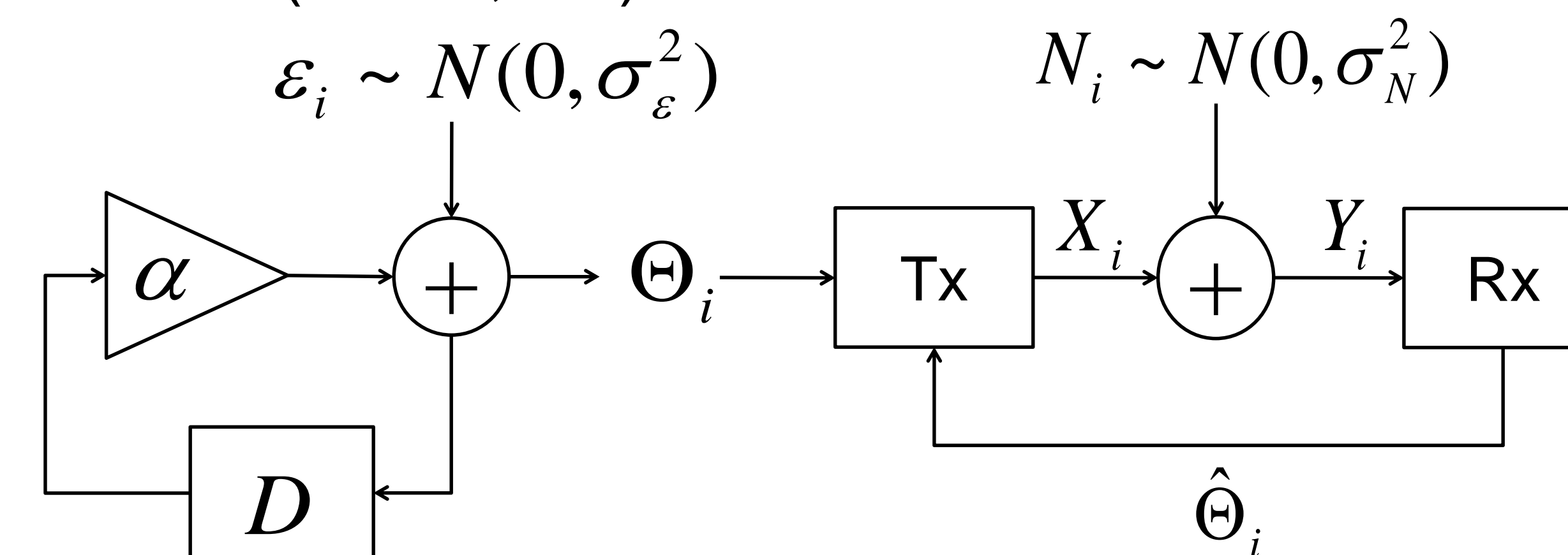
## Which GC-index is larger?

- Greater Granger-causal influence seen from the receiver to the sender.
- **The direction of greater Granger Causal influence can be opposite to the direction of flow of the message.**

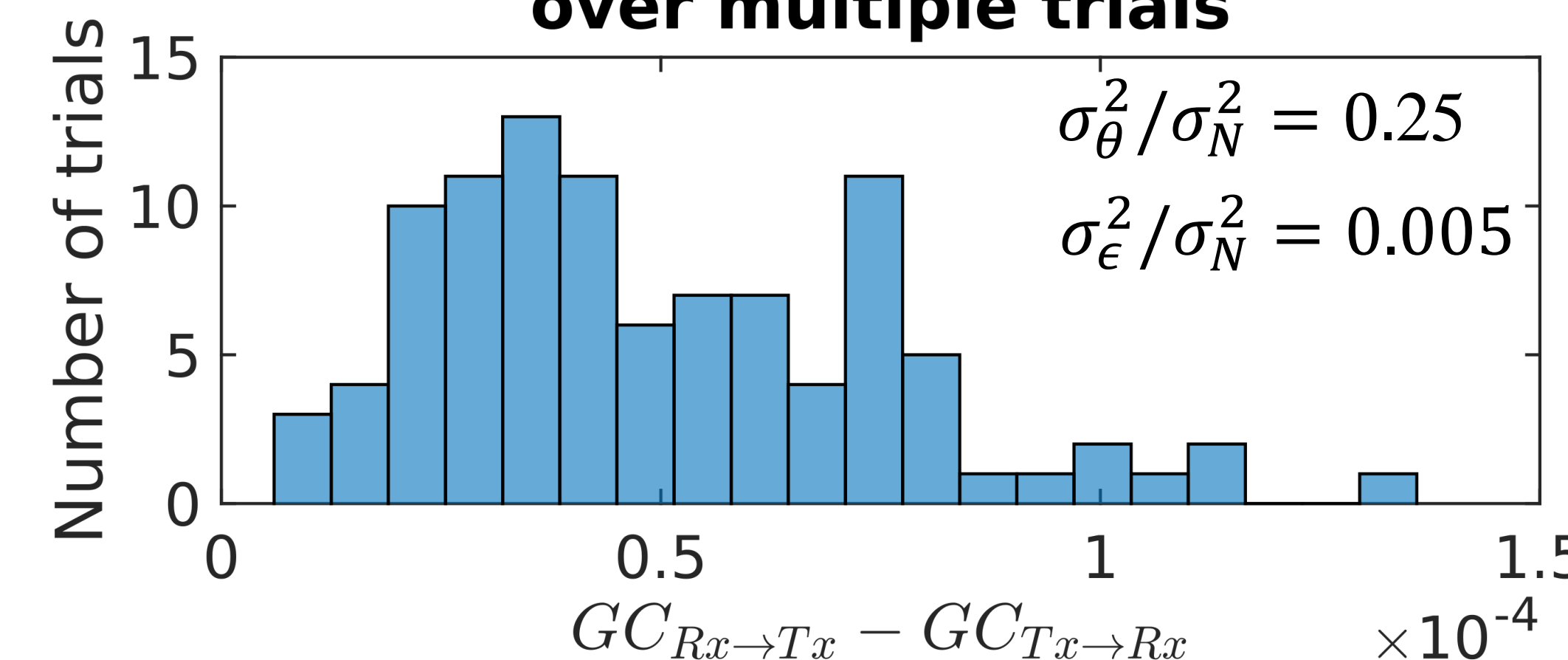


## Evolving source

- $\Theta$  evolves over time - all processes can be modeled as stationary AR processes.
- Corresponds better with MVAR models for Local Field Potentials.
- Simulates “Kalman filter”-type models of the brain (Grush, ‘04)



**Histogram of Granger causality difference over multiple trials**



**GC difference is always positive.**

## Conclusions

- Information-theoretically optimal feedback strategies exist, for which Granger-causal influence is opposite the true direction of information flow
- Previous critiques of Granger causality - the hidden node problem (Pearl, ‘00) and measurement noise (Friston et. al., ‘14) deal with incorrect estimation of the GC-index.
  - These can potentially be avoided by making better measurements.
- **Our work shows: even if GC-indices are estimated perfectly (i.e., no “hidden node” problem, and no measurement noise), the direction of larger GC index may not be the direction of information flow.**

## Acknowledgements

We would like to thank Nicola Elia and Rob Kass for useful discussions. Praveen Venkatesh is partially supported by a Dean’s Fellowship and a Presidential Fellowship.

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